Boundary conditions control in ORCA2

Eugene Kazantsev

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ORCA2 Configuration : $182 \times 149 \times 31$ nodes in curvilinear (x, y) coordinates with z levels.

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= \underbrace{v(\omega+f) - \frac{\partial (u^2+v^2)/2}{\partial x} - w\frac{\partial u}{\partial z}}_{\text{Advection}} - \underbrace{\frac{\partial A_u^h \xi}{\partial x} + \frac{\partial A_u^h \omega}{\partial y}}_{\text{Hor.Dissipation}} + \\
&+ \underbrace{\frac{\partial}{\partial z} A_u^z \frac{\partial u}{\partial z}}_{\text{Vert.Dissipation}} \underbrace{+ g\int_0^z \frac{\partial \rho(x,y,\zeta)}{\partial x} d\zeta}_{\text{Pressure gradient}} + \underbrace{g\frac{\partial (\eta + T_c\phi)}{\partial x}}_{\text{SSH}} \\
\frac{\partial v}{\partial t} &= -u(\omega+f) - \frac{\partial (u^2+v^2)/2}{\partial y} - w\frac{\partial v}{\partial z} - \frac{\partial A_u^h \xi}{\partial y} - \frac{\partial A_u^h \omega}{\partial x} + \\
&+ \frac{\partial}{\partial z} A_u^z \frac{\partial v}{\partial z} + g\int_0^z \frac{\partial \rho}{\partial y} dz + \frac{\partial (\eta + T_c\phi)}{\partial y} \\
\xi &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}, \qquad \omega = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \quad \text{Divergence, Vorticity} \\
w &= \int_H^z \xi(x,y,\zeta)d\zeta; \quad w(x,y,H) = 0 \quad \text{Vertical velocity} \\
\phi &= \frac{\partial \eta}{\partial t} \quad \text{Grav.Waves filter} \qquad\qquad A_u^h = const
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial T}{\partial t} &= \underbrace{-\frac{\partial uT}{\partial x} - \frac{\partial vT}{\partial y} - \frac{\partial wT}{\partial z}}_{\text{Advection}} \underbrace{+A_T^h\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)+}_{\text{Hor.diffusion}} \\
&+ \underbrace{\frac{\partial}{\partial z} A_T^z \frac{\partial T}{\partial z}}_{\text{Vert.diffusion}} + \text{Solar Radiation} + \text{Geothermal Heating} + \text{BBL} + \text{Surface} \\
\frac{\partial S}{\partial t} &= -\frac{\partial uS}{\partial x} - \frac{\partial vS}{\partial y} - \frac{\partial wS}{\partial z} + A_T^h\left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2}\right) + \frac{\partial}{\partial z} A_S^z \frac{\partial S}{\partial z} + \text{BBL} + \text{Surface} \\
A_T^z &= \text{Turbulent closure: } \sim max(A_0^z, C_k\ l_k\ \sqrt{\bar{e}}) \qquad (1) \\
\frac{\partial \bar{e}}{\partial t} &= A_v^z \left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] - A_T^z\, N^2 + \frac{\partial}{\partial z}\left[A_u^z \frac{\partial \bar{e}}{\partial z}\right] - c_\epsilon\, \frac{\bar{e}^{3/2}}{l_\epsilon} \\
\rho &= \rho(T,S), \quad N^2 = N^2(T,S), \quad R = R(T,S)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= \Big(S_x S_y v\Big) S_y(\omega + f) - D_x \frac{S_x u^2 + S_y v^2}{2} - S_z \Big(S_x w D_z u\Big) + D_x A_u^h \xi + D_y A_u^h \omega + \\
&+ g \int_0^z D_x S_z \rho(x,y,\zeta) d\zeta + D_{zz}(A_u^z u) + g D_x(\eta + T_c \phi) \\
\frac{\partial T}{\partial t} &= -D_x(u S_x T) - D_y(v S_y T) - D_z(w S_z T) + A_T^h \Big(D_x D_x T + D_y D_y T\Big) + \\
&+ D_{zz}(A_T^z T) + \text{Solar Radiation} + \text{Geothermal Heating} + \text{BBL} \\
\xi &= D_x u + D_y v, \quad \omega = D_y u - D_x v, \quad w = \int_H^z \xi(x,y,\zeta) d\zeta; w(x,y,H) = 0
\end{aligned}
$$

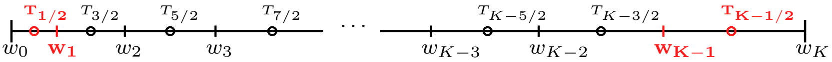
Interpolations and Derivatives

$$
\begin{aligned}
(Sw)_{k+1/2} &= \frac{w_{k+1} + w_k}{2} \; k = 1, \dots, K-1 \\
(DT)_k &= \frac{T_{k+1/2} - T_{k-1/2}}{h} \; k = 1, \dots, K-1
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= \Big(S_x S_y v\Big) S_y(\omega + f) - D_x \frac{S_x u^2 + S_y v^2}{2} - S_z \Big(S_x w D_z u\Big) + D_x A_u^h \xi + D_y A_u^h \omega + \\
&+ \quad g \int_0^z D_x S_z \rho(x,y,\zeta) d\zeta + D_{zz}(A_u^z u) + g D_x(\eta + T_c \phi) \\
\frac{\partial T}{\partial t} &= -D_x(u S_x T) - D_y(v S_y T) - D_z(w S_z T) + A_T^h \Big(D_x D_x T + D_y D_y T\Big) + \\
&+ \quad D_{zz}(A_T^z T) + \text{Solar Radiation} + \text{Geothermal Heating} + \text{BBL} \\
\xi &= D_x u + D_y v, \quad \omega = D_y u - D_x v, \quad w = \int_H^z \xi(x,y,\zeta) d\zeta; w(x,y,H) = 0
\end{aligned}
$$

Interpolations and Derivatives Modified Near the boundary

$$
\begin{aligned}
(Sw)_{k+1/2} &= \frac{w_{k+1} + w_k}{2}, \ k = 1, \dots, K-2, \ (Sw)_{1/2} = \alpha_0^S + \alpha_1^S w_0 + \alpha_2^S w_1 \\
(DT)_k &= \frac{T_{k+1/2} - T_{k-1/2}}{h}, \ i = 2, \dots, K-2, \ (DT)_1 = \alpha_0^D + \frac{\alpha_1^D T_{1/2} + \alpha_2^D T_{3/2}}{h}
\end{aligned}
$$

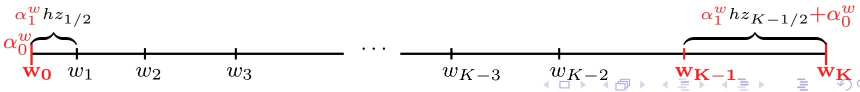


Space discretization

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= \Big(S_x S_y v\Big) S_y(\omega + f) - D_x \frac{S_x u^2 + S_y v^2}{2} - S_z \Big(S_x w D_z u\Big) + D_x A_u^h \xi + D_y A_u^h \omega + \\
&+ \quad g \int_0^z D_x S_z \rho(x,y,\zeta) d\zeta + D_{zz}(A_u^z u) + g D_x(\eta + T_c \phi) \\
\frac{\partial T}{\partial t} &= -D_x(u S_x T) - D_y(v S_y T) - D_z(w S_z T) + A_T^h \Big(D_x D_x T + D_y D_y T\Big) + \\
&+ \quad D_{zz}(A_T^z T) + \text{Solar Radiation} + \text{Geothermal Heating} + \text{BBL} \\
\xi &= D_x u + D_y v, \quad \omega = D_y u - D_x v, \quad w = \int_H^z \xi(x,y,\zeta) d\zeta; w(x,y,H) = \alpha_0(x,y)
\end{aligned}
$$

Vertical velocity

$$
\begin{aligned}
w_{i,j,K-1} &= \alpha_0^{w^b} - \alpha_1^{w^b} hz_{i,j,K-1/2} \xi_{i,j,K-1/2} \\
w_{i,j,k-1} &= w_{i,j,k} - hz_{i,j,k-1/2} \xi_{i,j,k-1/2} \quad \forall k : 2 \le k \le K-1 \\
w_{i,j,0} &= w_{i,j,1} + \alpha_0^{w^s} - \alpha_1^{w^s} hz_{i,j,1/2} \xi_{i,j,1/2}
\end{aligned}
$$



Vertical diffusion

$$\frac{\partial}{\partial z} A_u^z \frac{\partial u}{\partial z} \quad \text{is replaced by}$$

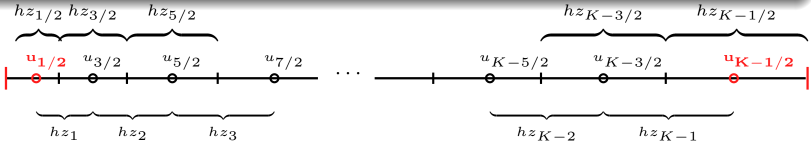
$$\left(D_{zz}u\right)_{i,j,1/2} = \frac{(A_u^z)_1}{hz_1 hz_{1/2}}\left(\alpha_2^{DzzU^s} u_{3/2} - \alpha_1^{DzzU^s} u_{1/2}\right)$$

$$\left(D_{zz}u\right)_{i,j,k-1/2} = \frac{1}{hz_{k-1/2}}\left(\frac{(A_u^z)_k}{hz_k}(u_{k+1/2} - u_{k-1/2}) - \frac{(A_u^z)_{k-1}}{hz_{k-1}}(u_{k-1/2} - u_{k-3/2})\right) \quad \forall k : 2$$

$$\left(D_{zz}u\right)_{i,j,K-1/2} = \frac{1}{hz_{K-1/2}}\left[\alpha_2^{DzzU^b}\frac{(A_u^z)_{K-1}}{hz_{K-1}} u_{K-1/2} - \alpha_1^{DzzU^b}\left(\frac{(A_u^z)_K}{hz_K} + \frac{(A_u^z)_{K-1}}{hz_{K-1}}\right) u_{K-3}\right.$$

$$\left.\frac{\partial u}{\partial z}\right|_{w0} = \alpha_0^{DzzU^s} + \frac{\tau_x}{hz_1\rho_0}, \quad \left.\frac{\partial v}{\partial z}\right|_{w0} = \alpha_0^{DzzU^s} + \frac{\tau_y}{hz_1\rho_0}, \quad \left.\frac{\partial T}{\partial z}\right|_{w0} = \left.\frac{\partial S}{\partial z}\right|_{w0} = \alpha_0^{DzzT^s}$$

$$u|_{bottom} = v|_{bottom} = \alpha_0^{DzzU^b} \quad T|_{bottom} = S|_{bottom} = \alpha_0^{DzzT^b} \tag{2}$$



$$
\begin{aligned}
\frac{u^{n+1}-u^{n-1}}{2dt} &= \underbrace{\gamma(u^{n+1}-2u^n+u^{n-1})}_{\text{Asselin filter}} - \left(S_x S_y v^n\right) S_y(\omega^n + f) - \\
&- D_x \frac{S_x(u^n)^2 + S_y(v^n)^2}{2} - S_z\left(S_x w^n D_z u^n\right) + \\
&+ \underbrace{D_x A_u^h \xi^{n-1} + D_y A_u^h \omega^{n-1}}_{\text{Explicit Euler}} + \underbrace{D_{zz} A_u^z u^{n+1}}_{\text{Impl.Euler}} + \\
&+ g\int_0^z D_x S_z \rho^n(x,y,\zeta)d\zeta + \underbrace{gD_x(\eta^{n+1} + T_c\phi^{n+1})}_{\text{Implicit}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{T^{n+1}-T^{n-1}}{2dt} &= \underbrace{\gamma(u^{n+1}-2u^n+u^{n-1})}_{\text{Asselin filter}} - D_x(u^n S_x T^n) - D_y(v^n S_y T^n) - \\
&- D_z(w^n S_z T^n) + \underbrace{A_T^h\left(D_{xx}T^{n-1} + D_{yy}T^{n-1}\right)}_{\text{Explicit Euler}} + \\
&+ \underbrace{D_{zz}(A_T^z)T^{n+1}}_{\text{Implicit}} + \text{Solar Radiation} + \text{Geothermal Heating} + \text{BBL}
\end{aligned}
$$

The models solution depend on initial conditions and a number of parameters:

$$\frac{\partial T}{\partial t} = -D_x(uS_xT) - D_y(vS_yT) - D_z^{(\alpha)}(wS_z^{(\alpha)}T) + A_T^h\left(D_{xx}T + D_{yy}T\right) + Dzz^{(\alpha)}(A_T^z)T$$

- The model $x(t) = \mathcal{M}_{0,t}(x_0, \alpha)$

We calculate the derivatives and their adjoints with respect to

$$x_0, \quad \alpha$$

by TAPENADE 3.6 (Tropics team, INRIA). that allows us

- to avoid a HUGE development/coding (a double of the classical one, at least)
- to obtain immediately the derivative with respect to any parameter we want.

TAPENADE 3.6 (Tropics team, INRIA) with the Memory usage optimization:

search for push/pop

```
CALL PUSHREAL8ARRAY(sold, nx*ny*nz)
CALL PUSHREAL8ARRAY(told, nx*ny*nz)
CALL PUSHREAL8ARRAY(vold, nx*ny*nz)
CALL PUSHREAL8ARRAY(uold, nx*ny*nz)
CALL PUSHREAL8ARRAY(ssh, nx*ny)
CALL PUSHREAL8ARRAY(s, nx*ny*nz)
CALL PUSHREAL8ARRAY(t, nx*ny*nz)
CALL PUSHREAL8ARRAY(v, nx*ny*nz)
CALL PUSHREAL8ARRAY(u, nx*ny*nz)
```

replace by

```
call push_uvts(u,v,t,s,ssh)
```

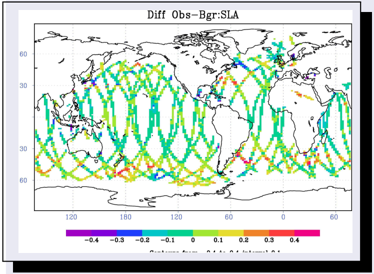
Procedure push/pop_uvts(u,v,t,s,ssh):

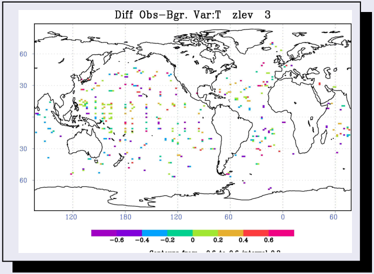
- does not push $n-1$ step and pops appropriate values (divides the required memory by 2)
- does not push u, v, t, s in lower level routines
- does not push values on continents (divides by 2)
- pushes values in Real*4 format (divides by 2)
- eventually pushes only odd timesteps and interpolate when poping (divides by 2)

Total reduction of required memory is up to 25 times.
10 hours window $\Longrightarrow$ 10 days window.

ECMWF data issued from Jason-1 and Envisat altimetric missions and ENACT/ENSEMBLES data banque.

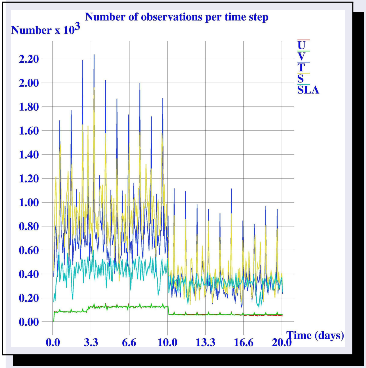
January, 1, 2006.



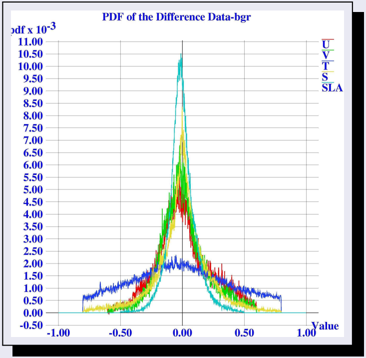


Difference between observations and background during the 1st of January.

January, 1-20 2006.



Number of observation per time step during the 1 – 20 Jan.2006



Probability density function for the difference (observation-background).

Cost function

The model: $x_N = \mathcal{M}_{0,N}(x_0, \alpha)$ with $x = (u, v, T, S, ssh)^T$

Cost function J

$$
\begin{aligned}
J &= \|x_0 - x_{bgr}\|^2_{B^{-1}} + \|\alpha - \alpha_{bgr}\|^2_{B^{-1}} + \\
&+ \sum_{n=0}^{N} t_n \|\mathcal{H}\mathcal{M}_{0,n}(x_0, \alpha) - y_n\|^2_{R^{-1}}
\end{aligned}
$$

Matrices: $B^{-1} = diag(10^{-4})$,
$R^{-1} = diag(1/\sigma_u, 1/\sigma_v, 1/\sigma_T, 1/\sigma_S, 1/\sigma_{ssh})$ where $\sigma_u^2 = \frac{1}{N_{obs}} \sum (u_{obs} - u_{bgr})^2$
Minimization is performed by M1QN3 (JC Gilbert, C.Lemarechal)

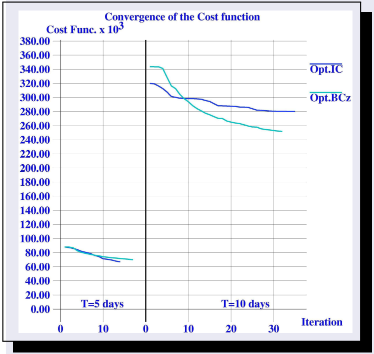
Data Assimilation – Forecast

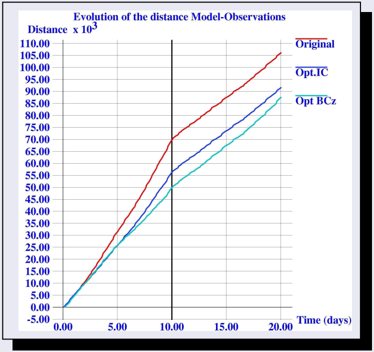
Assimilation window — 10 days,
Test time — 20 days.

The model: $x(t) = \mathcal{M}_{0,t}(x_0, \alpha)$ with $x = (u, v, T, S, ssh)^T$

$$\text{Distance: } \xi(t) = \sum_{n=0}^{t} \|\mathcal{H}\mathcal{M}_{0,n}(x_0, \alpha) - y_n\|_{R^{-1}}$$

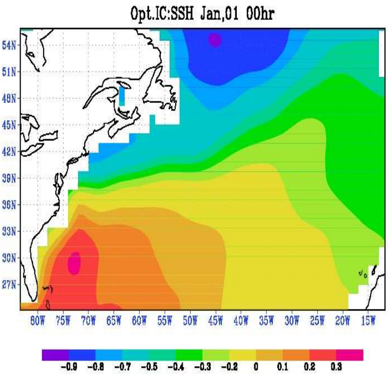
Convergence of J and evolution of ξ

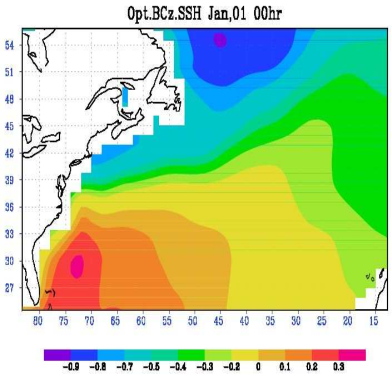




20 Cost function calls with $T = 5$ days and 40 calls with $T = 10$ days.

SSH, North Atlantic, January,1-30 2006.

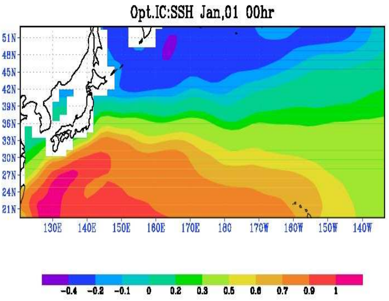


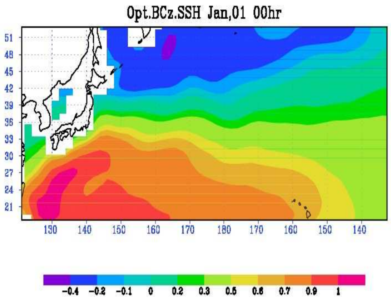


Optimal IC

Optimal BCz

SSH, North Pacific, January,1-30 2006.





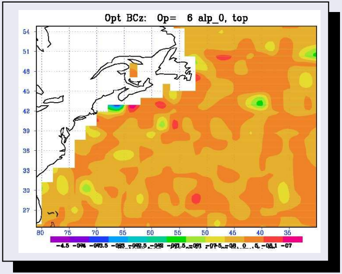
Optimal IC

Optimal BCz

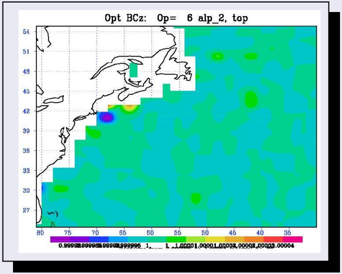
Modified formula

$$
\begin{aligned}
w_{i,j,K-1} &= \alpha_0^{w^b} - \alpha_2^{w^b} hz_{i,j,K-1/2}\xi_{i,j,K-1/2} \\
w_{i,j,k-1} &= w_{i,j,k} - hz_{i,j,k-1/2}\xi_{i,j,k-1/2} \quad \forall k : 1 \le k \le K-2 \\
w_{i,j,0} &= w_{i,j,1} + \alpha_0^{w^s} - \alpha_2^{w^s} hz_{i,j,1/2}\xi_{i,j,1/2}
\end{aligned}
$$

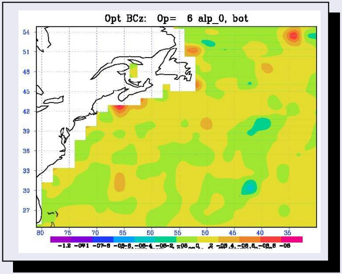
α for the vertical velocity w. North Atlantic.



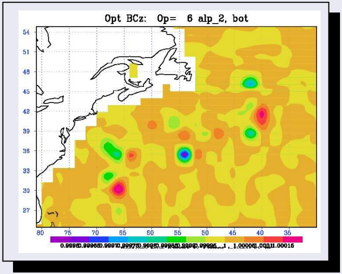
α_0 on the surface



α_2 on the surface

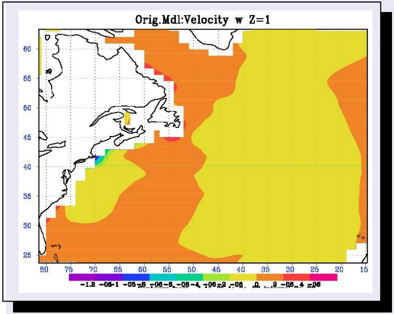


α_0 on the bottom

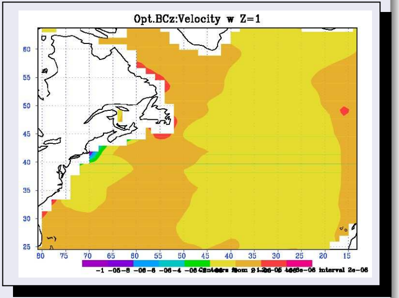


α_2 on the bottom

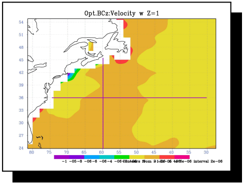
North Atlantic, January, 30, 2006, surface



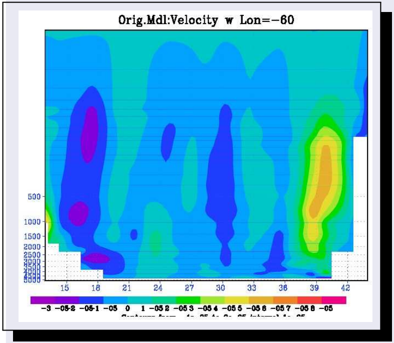
Original model



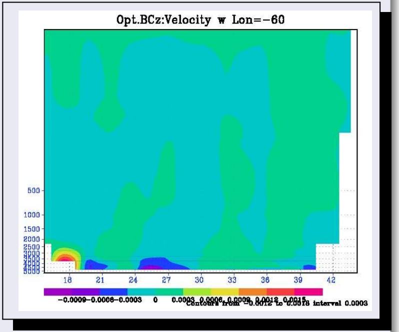
Optimal BCz



North Atlantic, January, 30, 2006, $y-z$ section

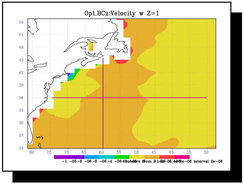


Original model

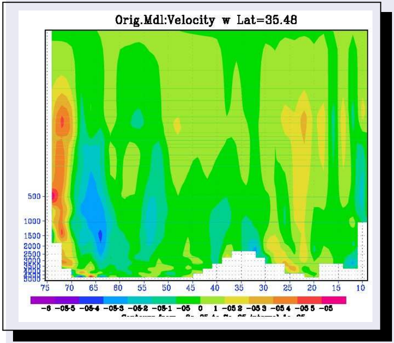


Optimal BCz

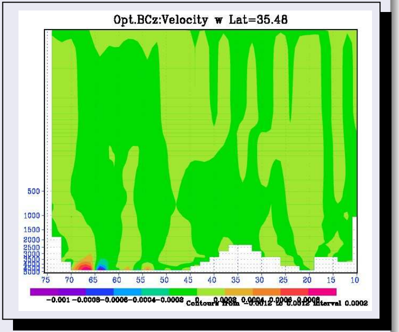
Vertical velocity



North Atlantic, January, 30, 2006, $x - z$ section

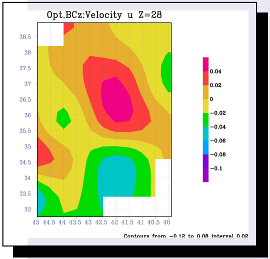


Original model

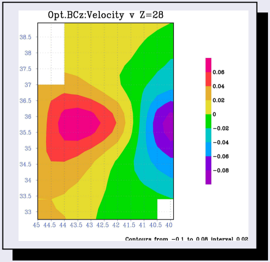


Optimal BCz

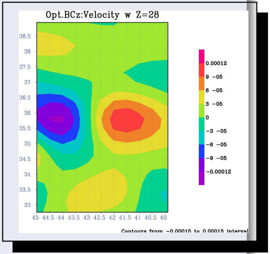
Levels $z = 28$ and $z = 29$



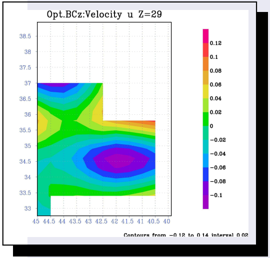
Velocity u



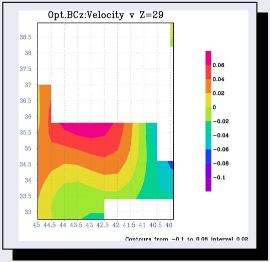
Velocity v



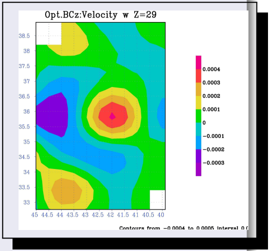
Velocity w



Velocity u



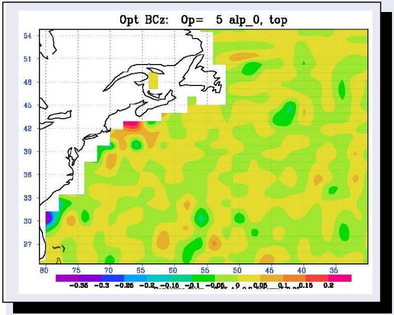
Velocity v



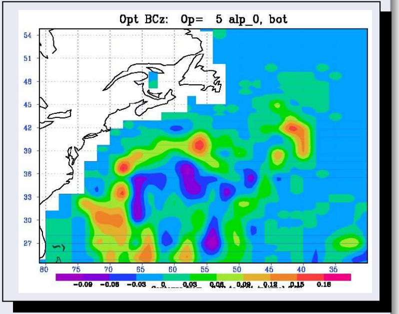
Velocity w

$$\left.\frac{\partial u}{\partial z}\right|_{w_0} = \alpha_0^{DzzU^s} + \frac{\tau_x}{hz_1\rho_0}, \quad \left.\frac{\partial v}{\partial z}\right|_{w_0} = \alpha_0^{DzzU^s} + \frac{\tau_y}{hz_1\rho_0},$$
$$u|_{bottom} = v|_{bottom} = \alpha_0^{DzzU^b}$$

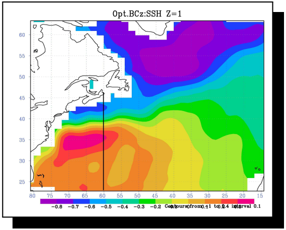
North Atlantic



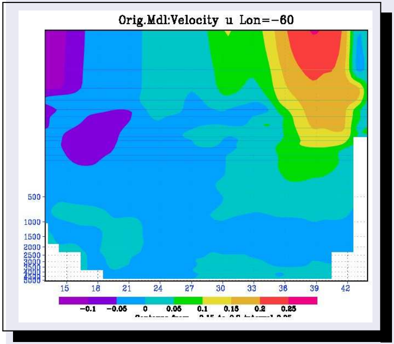
Surface



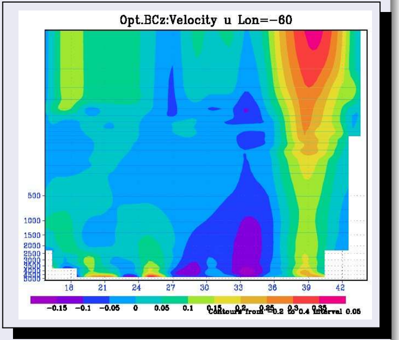
Bottom



North Atlantic, January, 30, 2006, Velocity u, $y - z$ section

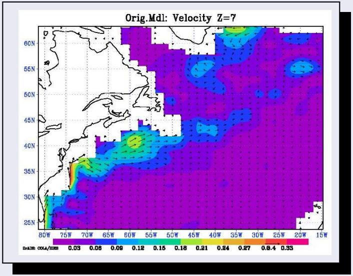


Original model

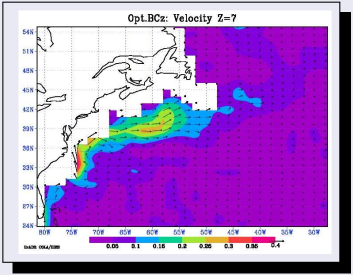


Optimal BCz

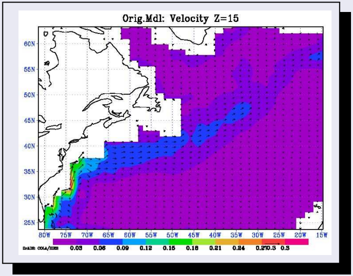
North Atlantic on the Jan.,30,2006



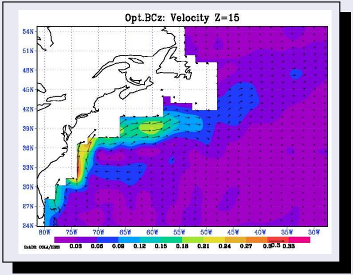
Original model, depth 65 m



Optimal BCz model, depth 65 m



Original model, depth 160 m



Optimal BCz model, depth 160 m

It is not an artefact.

North Atlantic

Modification of the SSH in the North Atlantic is strongly related to the boundary conditions of u and v especially on the bottom.

Boundary Conditions influence is important

- The cost function decreases more under BCz control than under IC control in the assimilation window
- The models forecast is closer to observations with optimal BCz than with optimal IC
- Stream jets are refined under BCz control

Tapenade allows us

- to generate TLM/AM almost immediately,
- to avoid a HUGE development/coding,
- to obtain immediately the derivative with respect to any parameter we want.

But, it requires more memory

	Resolution	TimeStep per day	Model Size	AM Size	Traj. size per 10 days
ORCA 2	$181 \times 149 \times 31$	15	650M	1.6G	660M
ORCA05	$722 \times 511 \times 46$	40	13G	33G	35G

BUT

As well as for any adjoint parameter estimation

- The control may violate the model physics;
- The physical meaning of α is difficult to understand;
- The set of α is not unique;
- The problem of identifiability is not addressed yet;
- The problem of stability is not even posed.

Consequently:

It is not a parameter estimation study, but

- a way to compensate model errors
- showing the most influent parameter (vertical BC for ORCA-2, lateral BC for Shallow-Water).

En vous remerciant pour votre attention

j'apprécierai beaucoup vos commentaires et critiques

http://www-ljk.imag.fr/membres/Kazantsev/orca2/index.html